Infrared and Raman Spectral Interpretations of the Ferroelectric Phases of $NaH_3(SeO_3)_2$ and $NaD_3(SeO_3)_2^*$

C. A. CODY,[†] R. C. LEVITT, AND R. K. KHANNA

Chemistry Department, University of Maryland, College Park, Maryland 20742

AND PHILIP J. MILLER

Department of Chemistry, University of Detroit, Detroit, Michigan 48221

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The infrared and Raman spectra of NaH₃(SeO₃)₂ and NaD₃(SeO₃)₂ have been recorded from 24 to 300°K. The interpretation, assignments, and analysis of the spectral studies are presented on the paraelectric α phase (proton disordered), ferroelectric β phase (proton ordered) and ferroelectric γ phase (proton ordered). A discussion of a newly proposed proton-triggered phase transition mechanism and a possible origin of the hydrogen-bonded O–H stretching region of KH₂PO₄-type ferroelectrics is given.

Introduction

Ferroelectric behavior in hydrogen-bonded crystals undergoing order-disorder phase transitions is believed to be triggered by ordered arrays of protons cooperatively moving from one side to the other side of the proton's double-minimum potential well (1-3). The high-temperature paraelectric phase (disordered phase) is believed to result from rapid proton tunneling between the two minima of this same potential well. Two of the betterknown crystal series displaying these features the alkali dihydrogen are phosphates, MH_2PO_4 , and the alkali trihydrogen selenites, $MH_3(\text{SeO}_3)_2$, where M = Li, Na, K, Rb, and Cs.

Although these general features of KDPtype ferroelectrics are supported by a variety of experimental techniques (4-10), an adequate description of the proton's role in triggering the order-disorder phase transition has not evolved. A consistent description of the origin of the hydrogen-bonded O-H stretching region of KDP-type crystals is also, in general, lacking.

We have recorded the infrared and Raman spectra of sodium trihydrogen selenite, and its deuterated analog, from 24 to 300° K. The results of this investigation have led to a suggestion of a new proton-triggering phase transition mechanism and a new interpretation of the hydrogen-bonded O-H stretching region in KDP-type crystals. These new interpretations are presented in this article. The authors also present their interpretation and assignments of the infrared and Raman spectra of NaH₃(SeO₃)₂ and NaD₃(SeO₃)₂.

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[†] Present address: NL Industries, Highlstown, N.J. 08520.

Experimental

Single crystals of $NaH_3(SeO_3)_2$ were grown by slow evaporation of aqueous solutions of Na_2CO_3 and SeO_2 , with a molar ratio of 1:4 in each case. For infrared studies, welldeveloped single crystals of $NaH_3(SeO_3)_2$ were dissolved in absolute methanol, and the resulting solution was deposited upon a 1-in.diameter KBr sample window and allowed to dry in a nitrogen atmosphere. The KBr sample window was placed in a Cryogenic Technology Inc. (CTI) Model 20 cold cell with a modified sample holder which consisted of a brass ring mounted directly to the cold state. Brass screw-in rings were used to secure the sample window, thus ensuring good thermal contact. The temperature was recorded on a CTI digital temperature indicator driven by a chromal-gold (0.07% iron) thermocouple mounted directly to the brass ring. Calibration of the digital readout was checked against liquid nitrogen, dry ice, and the ice point of water.

For small-temperature-variation experiments, a Fischer temperature controller (Model 22) in conjunction with two Yellow Springs' platinum resistor probes were coupled to the CTI cold cell. With this arrangement, temperatures could be recorded to $\pm 0.1^{\circ}$ K.

Infrared spectra were recorded on a Digilab FTS-14 spectrophotometer over the region 4000 to 450 cm⁻¹. All spectra were ratio plotted, with 100 scans of the cold cell and blank sample window used as a background spectrum and 100 scans of the cold cell and sample window used as the sample spectrum. All reported infrared spectra were scale expanded by the FTS-14 (i.e., the lowest actual transmission value was set at 0% and the highest actual transmission value was set at 100%) and were then plotted. Periodically, calibration of the FTS-14 was checked with a polystyrene film. Reported infrared frequencies are accurate to $\pm 2 \text{ cm}^{-1}$ for sharp bands and to ± 20 cm⁻¹ for broad bands.

Infrared spectra of deuterated compounds

were obtained in a similar manner with the exception of sample window preparation. Single crystals of $NaD_3(SeO_3)_2$ were mulled with nujol and deposited on the KBr windows as thin films. This procedure was used because deposition from CH₃OD solutions resulted in ~50% exchange of H for D via atmospheric water. Even the nujol method resulted in a small amount of exchange.

For Raman studies, large transparent crystals of NaH₃(SeO₃)₂ were oriented with a polarizing microscope and cut and polished into small cubes ~5 mm on an edge. The oriented cubes were mounted directly to the cold station with optical coupling compound. The optical coupling compound was mixed with powdered copper to ensure good thermal contact. Raman spectra of NaD₃(SeO₃)₂ were recorded in a similar manner except that unoriented single crystals were used as samples. This was done because deuterated crystals of a size sufficient to orient were not obtained from the growing solution.

Raman spectra were recorded on a Spex 1401 double monochromator coupled to a Coherent Radiation Model 52 argon ion laser (4880 A, ~500 mW) utilizing 90° scattering. A thin polaroid film was placed between the sample and slits of the monochromator to obtain the various polarizations. The monochromator was calibrated by scanning over nonlasing lines of the Model 52 laser. Reported Raman frequencies are accurate to ± 3 cm⁻¹ for sharp bands and to ± 20 cm⁻¹ for broad bands.

Discussion

A. Paraelectric Phase (a) of $NaH_3(SeO_3)_2$

The paraelectric phase of $NaH_3(SeO_3)_2$ (α phase) belongs to the space group $P_{2_{1/n}}$ containing two molecules per unit cell (11). Kaplan *et al.* (12) carried out a neutron diffraction study of $NaH_3(SeO_3)_2$ in the α phase and found the SeO_3^{-1} ions to be slightly distorted from an ideal pyramid (point group

 $C_{3\nu}$) resulting from differing bond lengths and angles of the molecule. Their determination of the SeO₃⁻ geometry was quite close to that reported in Vijayan's (11) X-ray study. Each of the above-mentioned structural works suggests a point group symmetry of SeO₃⁻ close to C_{s} , although the actual symmetry is C_1 .

Kaplan *et al.* (12) described the hydrogen bond network as consisting of SeO_3^{2-} ions hydrogen bonded to neighbouring SeO_3^{2-} so that each oxygen is hydrogen bonded to a different SeO_3^{2-} ion. This produces six SeO_3^{2-} ions hydrogen bonded in a closed loop. The proton in hydrogen bond O(1)-H(1)-O(1) (O...O distance, 2.60 Å) appears to be disordered and centered, and the proton in O(2)-H(2)-O(3) (O...O distance, 2.56 Å) appears to be situated closer to O(2) than to O(3); yet the possibility of a broad single-minimum potential well for H(2) is not ruled out. The numerals associated with the O and H atoms are crystallographic designations.

Soda and Chiba (9) employed the deuteron magnetic resonance technique to study the deuterium motions and hydrogen bond network above and below the phase transition in NaD₃(SeO₃)₂. They concluded that above T_c there are two nonequivalent hydrogen bonds and that the deuterium atoms in these bonds are tunneling rapidly between two sites in their respective double-minimum wells. Blinc et al. (13), on the basis of the results of a deuteron magnetic resonance study, reported NaD₃(SeO₃)₂ (designated a_D) to be isostructural with NaH₃(SeO₃)₂ in the roomtemperature phase.

Numerous infrared and Raman spectral studies have been reported for the various phases of $NaH_3(SeO_3)_2$ and $NaD_3(SeO_3)_2$ (4, 10, 14, 15). Khanna et al. (10) (infrared) were the first group to suggest that the 300- to 900-cm⁻¹ region of the α phase was very similar to the same spectral region crystalline Na₂SeO₃; therefore, they of concluded that all protons were disordered in the α phase of NaH₃(SeO₃)₂. Khanna et al. (10)further suggested that the lowtemperature ferroelectric phase transition was triggered by protons. They based their interpretation upon changes in the frequency of the SeO_3^{2-} vibrations as the transition was crossed.

Although later infrared and Raman studies also confirmed the proton's role in triggering the transition, many questions remained unanswered. In particular, previous spectral studies were not in agreement on the assignments of the internal modes, hydrogen-bonded O-H stretching and bending modes, and the degree of proton order in the different phases (10, 14, 15).

To clarify the situation, a study of the internal mode frequencies of $*SeO_3^{2-}$, $HSeO_3^{-}$,

426

700

689

359

332

408

686

675

349

327

| DEUTERATED ANALOGS ⁴ | | | | | | | | | |
|---------------------------------|----------------------------------|---------------------------------|--------------------------------|-------------------------------------|------------------|--|--|--|--|
| Assignment | *SeO ₃ ^{2-b} | HSeO ₃ ^{-c} | DSeO ₃ ^c | H_2 SeO ₃ ^d | D_2 Se O_3^d | | | | |
| $v_{i}(A')$ SeO(1) st | 800 | 602 | 584 | 832 | 832 | | | | |

408

867

798

320

359

408

863

801

316

359

TABLE I

The Stretching (st) and Bending (bd) Frequencies (cm⁻¹) of *SeO₃²⁻, HSeO₃⁻, and H₂SeO₃ and Their Deuterated Analogs^a

^a Reference (16).

 $v_2(A')$ SeO(2, 3) bd

 $v'_{3}(A')$ SeO(2, 3) st

 $v_1''(A'')$ SeO(2, 3) st

 $v'_4(A')$ SeO(1, 2, 3) bd

 $v_4''(A'')$ SeO(1, 2, 3) bd

^b SeO_{1}^{2-} with disordered protons in the lattice.

^c Frequencies from LiHSeO₃ and LiDSeO₃, respectively.

^d Frequencies from selenous acid (H₂SeO₃ and D₂SeO₃, respectively).

450

670

660

335

375

and H_2SeO_3 and their deuterated analogs in various alkali hydrogen selenite salts was undertaken (16). A summary of these results can be found in Table I.

These results indicated that the ordering of protons caused dramatic frequency shifts in the selenite modes so that the degree of proton order could be followed in this manner. Also, tentative assignments on the various $NaH_3(SeO_3)_2$ phases can be based on these findings.

From structure several the reports molecular units are possible in the α phase. If protons H(2) are ordered, the spectra would be representative of SeO_3^{2-} , $H_2SeO_3^{-}$, and $HSeO_3^-$ or H_2SeO_3 and SeO_3^{2-} . The simultaneous presence of three molecular units is possible if the H(2) protons order on different SeO_3^{2-} ions in one region and in the other regions of the crystal order on the same SeO_3^{2-} ions. In either case, ordered H(2) protons would produce H_2 SeO₃ frequencies in the spectra of the α phase.

If the H(2) protons are disordered, the spectra would be similar to $*SeO_3^{2-}$ and the

bandwidths would be broad because of intermolecular coupling. The lack of characteristic H_2SeO_3 stretching frequencies ($v_1 = 832$ cm⁻¹, $v'_3 = 700$ cm⁻¹, and $v''_3 = 689$ cm⁻¹) in the α phase infrared and Raman spectra of Figs. 1 and 2, respectively, in conjunction with the observation of strong broad *SeO₃²⁻-type stretching frequencies at $v_1 = 800$ cm⁻¹ and v'_3 and v''_3 at 660 cm⁻¹ in these same figures definitely indicates that all protons are dynamically disordered in the α phase of NaH₃(SeO₃)₂.

The structure reports indicate that the paraelectric α phase contains two distinct hydrogen bond lengths of 2.56 Å [O-H(2)-O] and 2.60 Å [O-H(1)-O] (11, 12). Two bending modes are observed in the infrared spectrum (Fig. 1). Making the assumption that the shorter the hydrogen bond length the higher the bending frequency leads us to assign the 1230-cm⁻¹ mode as an O-H(2)-O bend and the 1160cm⁻¹ mode as an O-H(1)-O bend.

Peercy (4) has assigned the ~ 1600 -cm⁻¹ mode as an O-H-O bend, but in consideration of the remarkable similarity of the proton



FIG. 1. The infrared spectrum of polycrystalline NaH₃(SeO₃)₂ (*a*-phase upper spectrum and β -phase lower spectrum).



FIG. 2. The b(cb)a Raman spectrum of a NaH₃(SeO₃)₂ single crystal in the *a* phase (300°K), β phase (103°K), and y phase (24°K).

stretching regions of the alkali dihydrogen phosphates and trihydrogen selenites we have assigned this mode as a proton stretch along with the 2760- and 2360-cm⁻¹ modes. The origin of these modes is discussed at length in the next section, entitled "Hydrogen Bond Stretching Region." The frequencies and assignments of the α and α_D phases are found in Table II.

B. First Ferroelectric Phase (β) of $NaH_3(SeO_3)_2$

Pepinsky and Vedam (5) have reported a phase transition in NaH₃(SeO₃)₂ occurring at 194°K. From 194 to 100°K, NaH₃(SeO₃)₂ is a ferroelectric triclinic crystal, space group P_1 (β phase) with eight molecules per unit cell resulting from a doubling of the α phase unit cell in the a and b directions (20). No X-ray diffraction studies have been reported on the β phase, although optical (17), dielectric (18, 19), nuclear magnetic resonance (20, 21), and specific heat (8, 22) data are known. NaD₃(SeO₃)₂ does not have an equivalent β_D low-temperature phase phase, the of $NaD_3(SeO_3)_2$, which has a transition temperature of 272°K, is monoclinic with space group Pm (γ_D phase) and is ferroelectric (20). The spectra of the γ_D phase of NaD₃(SeO₃)₂ are, therefore, discussed under the y phase of $NaH_{3}(SeO_{3})_{2}$

The authors feel that the $a \rightarrow \beta$ phase transition can be visualized as resulting from an

| TA | BL | Æ | Π |
|----|----|---|---|
|----|----|---|---|

| | _ | $NaH_3(SeO_3)_2$ | | | | | $NaD_3(SeO_3)_2$ unoriented ^a | | | | | |
|--|------|------------------|--------|--------|--------|-------|--|--------|--------------------|--------|--|--|
| Assignment | | Raman | | | | Raman | | | | | | |
| | ir | b(aa)c | b(ab)c | b(cc)a | b(cb)a | ir | pol. ₁ | depol. | pol. ₁₁ | depol. | | |
| Hydrogen bond | 2760 | 2710 | | 2780 | | 2070 | 2060 | | 2075 | | | |
| stretching region | 2360 | 2310 | | 2340 | | 1750 | 1760 | | 1750 | | | |
| | 1600 | 1600 | | 1600 | | 1250 | 1280 | | 1290 | | | |
| | | | | | | 980 | | | | | | |
| $\delta_{ m I}$ and $\delta_{ m 0}$ hydrogen | 1230 | 1200 | | 1250 | | 920 | | | | | | |
| bond bending | 1160 | | | | | | | 874 | | 875 | | |
| $v_1(A')$ *SeO ₃ ²⁻ | 820 | 803 | 796 | 803 | 801 | 830 | 813 | 812 | 812 | 813 | | |
| $v'_{3}(A')$ *SeO ₃ ²⁻ | 660 | 684 | 696 | 668 | 660 | | | | 700 | 696 | | |
| $v_{3}''(A'')$ *SeO ₃ ²⁻ | | | | | | 660 | 653 | 659 | | 657 | | |
| $\nu_2(A')$ *SeO ₃ ²⁻ | 425 | 443 | | | 448 | | 431 | 435 | 420 | 436 | | |
| $v_4(A'')$ *SeO ₃ ²⁻ | 359 | 391 | 385 | 385 | 386 | | | 381 | 375 | | | |
| $v'_4(A')$ *SeO ₃ ²⁻ | 324 | 337 | 345 | 331 | 331 | | 344 | 336 | 338 | 333 | | |
| OOv. stretching | 249 | 261 | 259 | 280 | 283 | | 250 | 249 | | | | |
| region | 242 | | | | | | | | | | | |
| Lattice region | | 182 | 163 | 186 | | | 158 | | 160 | | | |
| - | | 143 | | 140 | 139 | | 133 | | 133 | | | |
| | | 120 | 118 | 122 | 120 | | 110 | 113 | 110 | 113 | | |
| | | | | | | | | 94 | 93 | 92 | | |
| | | 74 | 73 | 73 | 74 | | 70 | 70 | 70 | 69 | | |
| | | | 62 | | | | | | | | | |

The Infrared and Raman Frequencies of the a and a_{D} , Phases

^a I and II are two different crystal orientations.

ordering of H(2) protons, with the remaining H(1) proton disordered. This point is discussed below.

Makita and Miki's (22) heat capacity data indicated $\Delta S = 1.03$ e.u. for the $\alpha \rightarrow \beta$ transition. The $\beta \rightarrow \gamma$ transition is accompanied by $\Delta S = 0.5$ e.u. or overall, the $\alpha \rightarrow \beta \rightarrow \gamma$ value for ΔS is 1.53 e.u. The magnitude of the reported phase transition entropy found in NaH₃(SeO₃)₂ and NaD₃(SeO₃)₂ indicates that these phase transitions are of the order-disorder type.

It is not unreasonable to attribute the entire phase transition entropy solely to changes in the proton position (23); therefore, if the ordering is complete for the $a_D \rightarrow \gamma_D$ transition $(\Delta S = 1.58 \text{ e.u.})$, then one expects each proton to contribute $\Delta S \simeq 0.5$ e.u. to the transition entropy. Evidence of total deuteron order in the γ_D phase has been presented by Soda and Chiba (9) (DMR), by Torrie and Knispel (14), (Raman), and by the present authors (see next section). The entropy argument would then imply that the $\alpha \rightarrow \beta$ transition entropy of 1 e.u. for NaH₃(SeO₃)₂ results from an ordering of two protons, leaving the third proton disordered.

Torrie and Knispel (14) previously indicated that the Raman spectra of the β phase suggest this phase to be more disordered than the γ phase. Their interpretation of the spectra was based upon the assumption that all protons in the β phase were disordered, but more ordered than in the α phase. They gave no explanation for a ferroelectric phase composed only of disordered protons. The β phase has two polarization directions. One direction is the *b* axis, and the other is in the *a*-*c* plane (17). As was pointed out in the discussion on the α phase, the ordering of H(2) protons could occur on the same or different SeO₃²⁻ ions throughout the crystal. The molecular units HSeO₃⁻, H₂SeO₃, and SeO₃²⁻ would then be present, and polarization reversal in both directions is realized by cooperative switching of protons on $HSeO_3^-$ and on H_2SeO_3 .

In the infrared and Raman spectra of the β phase of NaH₃(SeO₃)₂ (Figs. 1 and 2, respectively), frequencies characteristic of SeO₃⁻





FIG. 3. Temperature variations in the b(cb)a Raman spectrum of NaH₃(SeO₃)₂ in the $a \rightarrow \beta$ phase transition region.

TABLE III

| Assignment | ir | b(aa)c | b(ab)c ^a | b(cc)a | b(cb)a ^a |
|---|--------------|--------|---------------------|-------------|---------------------|
| Hydrogen bond stretching region | 2760 | | | 2760 | |
| | 2660 | | | | |
| | 2480 | | | | |
| | 2395 | 2340 | | 2380 | |
| | 1898 | | | | |
| | 1845 | | | | |
| | 1792 | | | | |
| | 1738 | 1655 | | | |
| | 1085 | 1055 | | 1600 | |
| | 1555 | 1550 | | 1000 | |
| δ_1 and δ_0 hydrogen bond bending | 1410 | | | | |
| | 1365 | | | | |
| | 1340 | | | | |
| | 1305 | | | | |
| | 1282 | | | 1290 | |
| | 1268 | 1245 | | | |
| | 1228 | 1198 | | 1235 | |
| | 998 | | | | |
| | 980 | 0.5.5 | 054 | 051 | 0.50 |
| | 960 | 955 | 954 | 951 | 952 |
| | 930 910 | | | 922 | |
| | <i>∫</i> 878 | | | | 876 |
| $V_1(A')H_2SeO_3$ | \860 | 863 | 867 | 865 | 864 |
| w/(4/)118-0- | ∫848 | 843 | | | |
| $V_3(A')$ HSeO ₃ | {829 | 825 | | | 830 |
| $v_3''(A'')$ HSeO ₃ and v_1 SeO ₃ ²⁻ | 805 | 798 | 795 | 798 | 799 |
| $v_{i}^{\prime}(A^{\prime})$ H_SeO, | { 748 | | 753 | 762 | 760 |
| · ;(1) / 12000 ; | (732 | (04 | 702 | 705 | 207 |
| | 695 | 694 | 703 | /05 | /0/ |
| $v_3''(A'')$ H ₂ SeO ₃ | 680 | | | 677 | 098 675 |
| | (670 | | | 660 | 015 |
| $v'(A')$ and $v''(A'')$ Se O^{2-} | 1650 | 651 | 655 | 655 | 659 |
| $V_3(A)$ and $V_3(A)$ (Sec V_3 | (050 | 0.51 | 000 | 639 | 007 |
| $v_{\star}(A')$ HSeO ₇ | 625 | | | 626 | 626 |
| 101 /1100 3 | (465 | 470 | 482 | 485 | 467 |
| $v_{2}'(A')$ HSeO ₁ | { | | | 4 77 | |
| | (| 466 | 451 | 451 | 453 |
| » (1/)H SeO | {439 | 430 | 431 | 433 | |
| V2(A)1125003 | (410 | 415 | | 410 | 415 |
| | 402 | 406 | 407 | | |
| $v_3(A')H_3SeO_3$ | 1200 | 205 | 393 | 200 | 200 |
| | 388 | 282 | 300 | 271 | 390 272 |
| | (311 | 515 | 515 | 5/1 | 515 |
| $v'_4(A')$ H ₂ SeO ₃ | 360 | 365 | 369 | | 368 |
| | (200 | | | | |

The Infrared and Raman Frequencies of the β Phase $(T = 113^{\circ}\text{K})$

| Assignment | ir | b(aa)c | b(ab)c ^a | | b(cb)a ^a |
|--|-------------------|--------|---------------------|-----|---------------------|
| | (354 | 352 | | 351 | 351 |
| $v_4^{\prime\prime}(A^{\prime\prime})H_2SeO_3$ | { 350 | | | | |
| | 346 | 339 | 340 | 340 | 340 |
| v'(A')HSeO — and SeO ² | <i>(</i> 338, 327 | | 330 | 330 | |
| A(II) IIBCO3 and BCO3 | { 323, 318 | | | 324 | |
| OOv_s stretching region | 304 | 306 | 306 | 308 | 309 |
| u - | 209, 280 | 283 | | 280 | |
| | 262 | 266 | 270 | 268 | 265 |
| | 254 | | 253 | 246 | |
| | 239 | | 241 | | 243 |
| | 215 | 207 | 228 | 211 | 229 |
| | 208 | | 213 | | |
| Lattice region | | 190 | 203 | 193 | 202 |
| U | | 184 | 181 | 183 | 181 |
| | | 167 | 167 | 170 | 171 |
| | | 162 | 164 | 166 | 166 |
| | | 156 | 154 | 152 | 153 |
| | | | 152 | | |
| | | 143 | 143 | 143 | 143 |
| | | 122 | 132 | 122 | 122 |
| | | | 123 | | |
| | | 115 | | 112 | 114 |
| | | 110 | 110 | | |
| | | 103 | 103 | 104 | 103 |
| | | 94 | 95 | 95 | 98 |
| | | 85 | 83 | 88 | 86 |
| | | 78 | 75 | 77 | 79 |
| | | 73 | 73 | 72 | 76 |
| | | 63 | 62 | 63 | 61 |
| | | 56 | 53 | 56 | 58 |
| | | 46 | 46 | 47 | |
| | | 43 | | | |
| | | 35 | | | |

TABLE III—Continued

^a The 1000- 3000-cm⁻¹ regions of these orientations were not recorded.

(800 cm⁻¹), HSeO₃⁻ (630 and 860 cm⁻¹), and H₂SeO₃ (830, 705, and 685 cm⁻¹) are seen, indicating that two protons have ordered. Torrie and Knispel's suggestion of residual disorder in the β can be explained by the presence of the remaining disordered third proton. This residual disorder in the crystal drives the bandwidths of all modes to intermediate values, thus giving the spectra the appearance of more order than the α phase, but less order than the γ phase.

Several groups have concluded that the $\alpha \rightarrow$

 β phase transition is second order (17, 18), whereas others have indicated that this transition is first order (20, 22). The Raman spectra recorded during the small-temperaturevariation experiments indicate that the $\alpha \rightarrow \beta$ phase transition is apparently second order. This conclusion is based upon the continuous spectral charges in the vicinity of T_c (Fig. 3) and the lack of thermal hysteresis upon cooling or heating the sample. The frequencies and assignments of the β phase are listed in Table III. C. Second Ferroelectric Phase (γ) of $NaH_3(SeO_3)_2$

Makita et al. (22) reported a second phase transition in $NaH_3(SeO_3)_2$ which occurred at about 100°K. Below 100°K, NaH₃(SeO₃), is monoclinic, space group Pm, with four molecules per unit cell, resulting from a doubling of the α phase unit cell in the adirection only (20). Optical (17), heat capacity (8), dielectric (14, 24), NMR (20, 21), and Raman (4, 14, 15) studies have been reported for the γ phase. NaD₃(SeO₃)₂ undergoes a phase transition at 272°K, from the a_p to the γ_D phase without the appearance of the β_D phase (13). Soda and Chiba's (9) DMR study below 272°K suggested the existence of asymmetric double-minimum potentials for the three hydrogen bonds connecting each SeO_3^{2-} ion. Complete ordering of all protons was also proposed for the γ_D phase.

The Raman spectra immediately below the $\beta \rightarrow \gamma$ transition (Fig. 2) is characterized by the appearance of intense HSeO₃⁻ bands replacing the β phase SeO₃²⁻ bands and by some rearrangement of the H₂SeO₃ frequencies. The rearrangement of H₂SeO₃ frequencies probably

results from changing Se–O bond lengths and force constants as the third proton orders. The appearance of HSeO₃⁻ bands and the corresponding disappearance of $*SeO_3^{2-}$ bands indicate that the remaining proton is ordered. The γ phase of NaH₃(SeO₃)₂ then contains ordered protons only.

In passing into the γ phase, the b component of polarization disappears (17). The a-cpolarization component of the β phase remains in the γ phase and retains its value as the transition is crossed (17), which is expected if the hypothesis of two ordered protons in the β phase is accepted. This results from a completely ordered network of protons. As protons are switched across the barrier they must all move into the opposite minimum, and since there are only two stable positions only one direction in the crystal can be ferroelectric.

Khanna et al. (10) recorded infrared spectra below the $\beta \rightarrow \gamma$ transition temperature, but they saw no spectral changes associated with a phase transition. Similarly, Torrie and Knispel (14) observed no changes in the infrared spectra at the $\beta \rightarrow \gamma$ transition. The authors repeatedly tried to detect $\beta \rightarrow \gamma$ phase transi-



FIG. 4. Comparison of the infrared transmission spectra of the β and γ_D phases of NaH₃(SeO₃)₂. (* denotes nujol bands.)



FREQUENCY (CM^{-1})

FIG. 5. Raman spectrum of unoriented NaD₃(SeO₃)₂ single crystal at 300, 270, and 24°K.

tions by using infrared spectroscopy. The samples were cycled around the transition temperature, and they were maintained 1 or 2° K below the $\beta \rightarrow \gamma$ transition for up to 24 hr, to allow for a sluggish phase change; all attempts, however, were unsuccessful.

It is possible that these differences in the infrared and Raman spectra are due to different sample forms, i.e., polycrystalline versus single crystal, but this would seem to be eliminated by the work of Torrie and Knispel (14), who observed the $\beta \rightarrow \gamma$ transition in the Raman spectra recorded off of a pellet pressed from powdered NaH₃(SeO₃)₂.

To complicate matters, the β phase of NaH₃(SeO₃)₂ and the γ_D phase of NaD₃(SeO₃)₂ yield similar infrared spectra (Fig. 4); yet Raman spectra of the β and γ_D phases are quite different (Figs. 2 and 5, respectively). Before an interpretation of this

feature can be given, additional research must be carried out.

The low-frequency Raman spectra for the $\beta \rightarrow \gamma$ transition is presented in Fig. 6. The transition is thought to be first order based upon the discontinuous spectral changes at T_c . A large thermal hysteresis of approximately 15.4°K was noticed upon heating the sample in the γ phase.

The frequencies and assignments of the γ and γ_D phases can be found in Table IV.

Mechanism of the Transition

To discuss the mechanism of the phase transition from the disordered to the ordered state, an examination of the proton's energy level scheme in the double-minimum potential with a low intrabond barrier is in order. The proton's energy system can be described as

TABLE IV

| | | NaH ₃ (SeO ₃) ₂ | | | | | $NaD_{3}(SeO_{3})_{2}^{a}$ unoriented ^b | | | | | |
|--|-------------|---|--------|--------|------------|------------|--|--------|--------------------|--------|--|--|
| | | Raman | | | | 24°K Raman | | | | | | |
| Assignment | ir | b(aa)c | b(ab)c | b(cc)a | b(cb)a | ir | pol. ₁ | depol. | pol. ₁₁ | depol. | | |
| Hydrogen bond | 2760 | 2710 | | 2780 | | 2070 | 2090 | | 2070 | | | |
| stretching | 2660 | | | | | | 2020 | | | | | |
| | 2480 | | | | | 1850 | 1855 | | 1855 | | | |
| | 2395 | 2350 | | 2400 | | 1760 | 1770 | | 1765 | | | |
| | 1898 | | | | | | | | | | | |
| | 1845 | | | | | 1580 | | | | | | |
| | 1792 | | | | | | | | | | | |
| | 1738 | 1600 | | | | 1000 | 1200 | | 1204 | | | |
| | 1085 | 1050 | | 1590 | | 1285 | 1300 | | 1294 | | | |
| | 1595 | 1520 | | 1580 | | 1220 | 1252 | | 1220 | | | |
| | 1410 | 1528 | | | | 1140 | 1155 | | 1150 | | | |
| | 1365 | | | 1375 | | | 1025 | | | | | |
| å and å hydrogan | 1240 | | | 10.12 | | 1009 | 1005 | | | | | |
| bond bending | 1340 | | | 1310 | | 085 | 088 | | | 088 | | |
| bond bending | 1282 | 1250 | | 1275 | | 965 | 300 | | | 900 | | |
| | 1202 | 1250 | | 1215 | | 740 | | 930 | 930 | 702 | | |
| | 1268 | 1230 | | | | 978 | | - | | | | |
| | 1200 | 1190 | | 1230 | | 899 | 897 | | 892 | 892 | | |
| | 988 | 1170 | 988 | 1200 | | 611 | 617 | 617 | 618 | | | |
| | 980 | | 985 | | | 602 | 604 | 603 | 600 | | | |
| | 960 | 952 | , | | | 583 | 590 | 598 | 583 | | | |
| | 910 | 912 | 911 | 912 | 910 | 522 | 531 | 530 | 520 | 522 | | |
| (())) | (878 | 872 | | | | 870 | 874 | 873 | 881 | 878 | | |
| $v_1(A')H_2SeO_3$ | 860 | | | | | 862 | | 864 | 867 | 868 | | |
| | 848 | | 842 | 852 | 851 | | | | | | | |
| v!(A')HSeO, | { | | | | | | | | | | | |
| · ;(-) ; | (829 | 838 | 827 | | 830 | 825 | 838 | 837 | 010 | | | |
| $v_3''(A'')$ HSeO ₃ | 805 | 704 | 800 | 705 | 705 | 805 | 80/ 709 | 807 | 810 | 702 | | |
| | 603 (749 | 790 | 762 | 750 | 740 | 805 | 190 | 100 | | 193 | | |
| $v'_3(A')H_2SeO_3$ | 1740 | //4 | /05 | 730 | 740 | 705 | 700 | 700 | 708 | 706 | | |
| | (605 | 701 | | 720 | 750 | 688 | 688 | 692 | /08 | 692 | | |
| $v_3''(A'')$ H ₂ SeO ₃ | 680 | 693 | 700 | 694 | 692 | 000 | 000 | 688 | | 0,5 | | |
| | (670 | 070 | 100 | | 072 | 670 | 674 | 676 | | 673 | | |
| $v_{1}(A')$ HSeO ₅ | {650 | 660 | 655 | 651 | 667 | 660 | 660 | 660 | 661 | 662 | | |
| 16- , 3 | 625 | 625 | 630 | 628 | 623 | 630 | 640 | 643 | 643 | 640 | | |
| (10100 0 | (465 | 455 | 453 | 452 | | | | | | | | |
| $v_2(A')$ HSeO $\frac{1}{3}$ | { | | 436 | 439 | 440 | | 435 | 431 | 435 | 431 | | |
| | • | | | | | | 431 | | | | | |
| $v_2(A')H_2SeO_3$ | 5 | 403 | 419 | 410 | 410 | | 406 | 401 | 413 | 412 | | |
| | l | | 404 | | | | 403 | | | 402 | | |
| $v_{i}^{\prime\prime}(A^{\prime\prime})$ HSeO ₇ | 5 | 202 | 391 | 388 | 386 | | 386 | 385 | 386 | 384 | | |
| | l | 382 | 383 | 366 | 380 365 | | 372 | 371 | | 365 | | |
| $v_4 (A^{-}) \Pi_2 SeU_3$ | | 303 | 545 | 200 | 202 | | 333 | 300 | | 505 | | |

The Infrared and Raman Frequencies of the γ and $\gamma_{\rm D}$ Phases ($T=24\,^{\circ}{\rm K})$

| | NaH ₃ (SeO ₃) ₂ | | | | | $NaD_3(SeO_3)_2^a$ unoriented ^b | | | | |
|--|---|--------|--------|--------|--------|--|------------|--------|--------|--------|
| | | Raman | | | | | 24°K Raman | | | |
| Assignment | ir | b(aa)c | b(ab)c | b(cc)a | b(cb)a | ir | pol. | depol. | pol.11 | depol. |
| $v'_4(A')$ H ₂ SeO ₃ | | 352 | 350 | 352 | 352 | | 352 | 341 | 341 | 348 |
| | | (330 | 338 | 330 | 328 | | 332 | 333 | 333 | 330 |
| $v'_4(A')$ HSeO ₃ | | 1 | | | | | 327 | 327 | | |
| | | (305 | 313 | 305 | 300 | | 301 | 301 | 301 | 301 |
| | | | | 291 | 290 | | 291 | 291 | | 290 |
| OOv_s stretching | | | 283 | 286 | 282 | | 270 | 270 | 270 | |
| region | | 259 | 258 | 259 | 255 | | 256 | 257 | 251 | |
| | | 235 | 235 | | 230 | | 241 | 242 | | 243 |
| | | 223 | 225 | 225 | 223 | | 224 | 225 | 220 | 220 |
| Lattice region | | 193 | 194 | | 191 | | 198 | 203 | | |
| | | 183 | 183 | | | | 191 | 200 | 190 | 196 |
| | | | | | | | | 191 | | |
| | | 173 | 174 | | 177 | | 178 | 182 | 171 | 171 |
| | | | 171 | | | | | 178 | | |
| | | 163 | 164 | 165 | 165 | | 164 | 163 | 161 | 161 |
| | | 156 | 158 | 156 | 157 | | 157 | 159 | | |
| | | 150 | 150 | | 150 | | 148 | 150 | 149 | 149 |
| | | 143 | | 145 | 144 | | 141 | 145 | 142 | |
| | | | | | | | | 141 | | |
| | | 140 | 141 | 149 | 149 | | 132 | 133 | 138 | 136 |
| | | 132 | 130 | 130 | 130 | | | | 131 | |
| | | 122 | 120 | 120 | 121 | | 121 | 123 | | |
| | | 118 | 112 | | 119 | | 114 | 117 | 119 | 118 |
| | | 103 | 100 | 107 | 105 | | 106 | 103 | 103 | 100 |
| | | | | 96 | 97 | | 98 | 98 | 98 | |
| | | 92 | 92 | 92 | 91 | | 94 | | | |
| | | | 83 | | 86 | | 88 | 89 | 88 | 87 |
| | | | 80 | | | | 82 | 82 | | |
| | | 75 | 74 | 76 | 75 | | 76 | 76 | 75 | 74 |
| | | 64 | 62 | 67 | 65 | | 63 | 62 | 64 | 69 |
| | | 59 | | 58 | 58 | | 58 | 58 | 55 | 62 |
| | | 50 | 53 | 50 | 49 | | 52 | 52 | 50 | 50 |
| | | 46 | 46 | | | | | 48 | | |

TABLE IV-Continued

^{*a*} In the assignment column replace H with D, i.e., $v_1(A')H_2SeO_3 \rightarrow v_1(A')D_2SeO_3$.

^b I and II are two different directions of the unoriented crystal.

consisting of a series of discrete levels, each level having its own distinct probability and consequently, frequency, of tunneling. The lowest level has a tunneling frequency of zero, and consequently, the proton can be viewed as confined to one side of the well. Each succeeding level has an increasingly higher tunneling probability, until in the vicinity of the barrier top the probability of proton tunneling becomes nearly 1. Here, the proton tunnels faster than the frequencies of the rest of the lattice, owing to its smaller mass. For convenience, the treatment below assumes only the existence of a ground state (the nontunneling level) and one other state right below the intrabond barrier (the fast tunneling level).

Three important restrictions on this description must be noted. First, the intrabond barrier is quite small, ~600 cm⁻¹ above the ground state (25). Second, the separation between the slow and fast tunneling levels could be as small as 50 to 200 cm⁻¹. This assumption is made because the fast tunneling level must be populated at moderate temperatures (~150–300°K) to account for the disordered phase. Third, the fast tunneling of protons is a cooperative phenomenon only within very small regions of the crystal.

The disordered phase of the crystal can now be described as having the majority of its protons thermally populating the fast tunneling level, producing a symmetric proton density and consequently a more symmetric crystal structure. The SeO_3^{2-} ions view these protons in average positions, and the spectra are characteristic of *SeO_3^{2-} ions with broad internal modes. The ordered phase of the crystal consists of the majority of protons in the nontunneling ground level; the SeO_3^{2-} ions view these protons as confined to one side of the well, producing characteristic HSeO_3^{-} or H_2SeO_3 frequencies.

The "critical" slowing down of proton tunneling in the immediate vicinity of the ordered phase, as reported by NMR experiments (25), arises from a rapid shift in proton population between the levels. The tunneling frequencies in such cases represent the numerical average between the number of protons in the fast tunneling and nontunneling levels. Thus, the thermal population of levels is described and controlled by (at extreme temperature levels) a Boltzmann-like distribution function. Far above or below T_c a "normal" population is predicted; however, the cooperative nature of the tunneling motion causes a rapid shift in population near T_c , i.e., a cascade. The temperature spread of a second-order phase transition is directly gov-



FIG. 6. The b(cb)a Raman spectrum of the lattice region of NaH₃(SeO₃)₂ in the $\beta \rightarrow \gamma$ phase transition temperature region.

erned by the energy separation of the levels and by the magnitude of the interaction between static and tunneling protons.

This approach to a theory of phase transitions in KDP-type crystals appears to be quite simple because the exact form of the coupling between tunneling protons, coupling between static and tunneling protons, and the temperature dependence of protons cascading between energy levels has been ignored. Inclusion of these terms in the model is extremely difficult because of the lack of experimental data. This model, therefore, is simplistic in its approach but is attractive because it correctly predicts many of the essential experimental facts.

In NaH₃(SeO₃)₂ protons occupy two different hydrogen bonds. Thus, the energy separation between the fast and nontunneling levels is expected to be different for each distinct hydrogen bond, and, therefore, from this description two different proton-triggered phase transitions are expected.

Spectroscopic evidence in support of the discrete energy level proposed here is based upon the behaviour of the hydrogen-bonded *SeO $_{3}^{2-}$ modes as a function of temperature. Upon passing into the ordered phase, if each proton has a temperature-dependent tunneling frequency, which slows down as the temperature is lowered, then the changing perturbation will continuously shift the $*SeO_3^{2-}$ frequencies until $HSeO_3^-$ or $H_2SeO_3^-$ frequencies dominate. However, as seen in Fig. 3, as one crosses the order-disorder transition, the intensities of the $*SeO_3^{2-}$ modes simply decrease, whereas those of the $HSeO_3^-$ modes increase. The spectra never indicate that proton tunneling has slowed down; quite the contrary is true: The spectra indicate that at a temperature of 193.3°K the crystal has some proton population in the fast tunneling level and some proton population in the static level.

Additional support for this point comes from the behavior of the spontaneous polarization with temperature. For second-order phase transitions, the polarization continuously increases, below T_c , as the temperature decreases. This result is due to the buildup of the ground level population, and in fact the final "saturated" polarization value represents a crystal with nearly the entire population of protons resting in the ground state.

At present, the actual shape of the proton potential well is an ambiguous question. Neutron and X-ray diffraction, along with NMR and DMR experiments, suggest that in many KDP-type crystals the well is a symmetric double-minimum type. However, the same experimental interpretation can be derived on the basis of an asymmetric well if the rapid proton tunneling averages out the asymmetry in the disordered phase. We favor the asymmetric case since it readily accounts for the above-described mechanism of the transition and yet removes the unnecessary assumption of a symmetric double minimum.

The well-known deuterium temperature shift for a proton-triggered phase transition is also readily explained by this model. The original fast tunneling level will drop down into the well upon deuteration and become a nontunneling level by virtue of the fact that it now lies quite far below the intrabond barrier top. To populate a fast tunneling level (say the third or fourth level above the ground state) to form the disordered phase, a much higher temperature must be reached.

Hydrogen Bond Stretching Region

The presence of an intrabond barrier in KDP-type crystals $(MH_2PO_4, MH_2AsO_4, MH_3(SeO_3)_2$, etc., where M = Li, Na, K, Cs, and Rb) has been ussed as the main criterion controlling the number, frequency, and breadth of the hydrogen-bonded O-H stretching bands of these compounds (26, 27). However, several valid criticisms of these interpretations can be advanced. For instance, to calculate frequencies close to those observed in KH₂PO₄ (2800, 2400, and 1750 cm⁻¹) the intrabond barrier must be ~3300 cm⁻¹ above the ground state (26). An intrabond

barrier of this magnitude eliminates any possibility of an order-disorder phase transition at moderate temperatures $(50-300^{\circ}K)$. Furthermore, the calculated frequencies are in poor numerical agreement with those observed, and also, typical measured intrabond barriers are generally about 600 cm⁻¹ (25). These calculations are also inadequate in their prediction that at least one mode's intensity is temperature dependent.

Perhaps the severest criticism of these approaches comes from the realization that KDP-type crystals displaying a wide variance in hydrogen bond lengths, geometries, and phase transition behavior should have corresponding wide variances in calculated O-H stretching frequencies, intensities, and bandwidths, when in fact these same compounds have remarkably similar stretching regions (28).

Because of these difficulties, we believe that two nearly independent processes, one controlling the phase transition and another controlling the structure of the O-H stretching region, are responsible for these effects.

Schmidt and Uelhig (25) were the first to note that two independent proton tunneling processes were occurring in KD₂PO₄: a slow interbond tunneling (diffusion between hydrogen bonds) which was unaffected upon crossing T_c , and a fast intrabond tunneling (within a hydrogen bond) which critically slowed down as T_c was crossed. It is plausible, therefore, that the barrier controlling proton diffusion controls the number, breadth, and frequencies of the O-H stretching modes, and that the intrabond barrier controls the mechanism and nature of the phase transition. This description is very reasonable in view of the experimental data: First, the diffusion barriers of KDP-type crystals are similar in magnitude $(4000-6000 \text{ cm}^{-1})$ (29), and thus the structure of the O-H stretching region of these crystals should be similar. Second, the diffusion barrier top lies in close proximity to an excited state and, therefore, may result in a large splitting of states which could explain the observed multiplicity. Because the large splitting occurs in an excited state no thermal depopulation (which would cause intensity changes) of these levels is anticipated upon cooling. Third, a normal deuteration shift of $v_D/v_H = 0.707$ is predicted and has been observed in KDP-type crystals. Fourth, the lack of any significant change in frequency, intensity, or bandwidth of the O-H stretching modes upon crossing below T_{cr} regardless of the transition type, arises because of the independence of the diffusion barrier with respect to the phase transition. Fifth, the abnormal bandwidths of these modes are due to long-range well-to-well diffusion coupling. Along these lines, previous researchers have noted that low concentrations ($\sim 1-5\%$) of deuterated species in their undeuterated hosts lead to sharp O-D bands (30-32). At higher concentrations, the bands markedly broaden, indicating the onset of coupling. This illustrates that the large breadth of these bands in the spectra of pure compounds cannot be due to the dynamics of individual wells but must arise from well-to-well coupling.

A more elaborate discussion of the diffusion barrier O-H stretching region hypothesis can be found in an earlier publication (33).

Conclusion

The infrared and Raman spectra of the ferroelectric phases of $NaH_3(SeO_3)_2$ and $NaD_3(SeO_3)_2$ are extremely complicated because of the large number of molecules in the unit cell and the low symmetry of each phase. The assignments presented in this article are of a tentative nature and should be refined by a complete study of the infrared and Raman spectra of H₂SeO₃, NaHSeO₃, and deuterated analogs. An understanding of the spectral properties of these compounds would greatly aid in assigning the internal and lattice mode region of $NaH_3(SeO_3)_2$ and determining the frequency, polarization properties, and atomic motions of the tunneling normal mode. Studies of this nature are presently being carried out.

The nonexistence β_D phase of NaD₃(SeO₃)₂ illustrates the changing hydrogen bond dynamics upon deuteration. Under the proposed phase transition mechanism the fast tunneling level drops down farther into the potential well and becomes nontunneling, because of the increased hydrogen bond length and deuteron mass. Offsetting these factors are variations in deuteron coupling efficiency in the new fast tunneling levels at higher energy. These factors working in combination could readily account for the anisomorphism of the NaH_xD_{3-x}(SeO₃)₂ solid solution series.

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